Original scientific paper UDK 691.32:624.04
624.04:691.32

EXPERIMENTAL ANALYSIS OF STRUCTURE RESPONSE TO NON-UNIFORM SUPPORT EXCITATION

EKSPERIMENTALNA ANALIZA ODZIVA GREDNOG NOSAČA NA NEJEDNOLIKU POBUDU OSLONACA

Simona Dobrilla*, Nina Čeh*, Miran Tuhtan*, Gordan Jelenić*

Abstract

Experimental testing of the dynamic response of a simply supported beam model with discrete masses has been carried out. The range of excitation amplitudes for which the response is linear have been determined. The experimentally obtained displacements are presented for the case of harmonic uniform and non-uniform excitation, as well as for the simulation of the Northridge earthquake, which could be used as experimental benchmarks for multiple support excitation.

Key words: *dynamic analysis, non-uniform earthquake excitation, shake tables, optical displacement measurement, beam structures*

Sažetak

Provedeno je eksperimentalno ispitivanje dinamičkog odziva modela slobodno oslonjene grede s dodatnim koncentriranim masama. Određene su amplitude pobude unutar kojih je odgovor grede linearan. Prikazani su eksperimentalno dobiveni rezultati za pomake grede prilikom harmonijske jednolike i nejednolike pobude te prilikom simulacije Northridge potresnog zapisa, koji se mogu koristiti kao eksperimentalni benchmark primjerci za nejednoliku pobudu oslonaca.

Ključne riječi: *dinamička analiza, nejednolika potresna pobuda, potresne platforme, optičko mjerenje pomaka, gredne konstrukcije*

*University of Rijeka, Faculty of Civil Engineering, Radmile Matejčić 3, 51000 Rijeka
E-mail: simona.dobrilla@gmail.com, {nina.ceh,miran.tuhtan,gordan.jelenic}@uniri.hr

Rad zaprimljen: 26. 1. 2018. Rad prihvaćen: 12. 3. 2018.

1. Introduction

In regions where earthquakes are prevalent, the main concern is determining the structural response to earthquake-induced motion of the base of the structure. Unlike uniform excitation, where all the supports undergo an identical ground motion and move as one rigid base, when non-uniform excitation is applied for the same travelling seismic wave, each support will receive different ground motion input [1].

For long-span structures, such as bridges and dams, or structures built on significantly varying soil types, earthquake induced multi-support excitation may greatly affect overall dynamic response of structures. This introduces a response case, which is usually not taken into account in the assessment of dynamic response of structures. In the extreme case, this could lead to failure due to excessive relative displacements of the supports.

During the past 30 years, many experts in the field of structural dynamics, such as Chopra [2] and Clough & Penzien [3], to mention only a couple of well-known sources, have been studying the response of structures under spatially variable ground movement and provided theoretical background, along with the solution methods for the case of multi-support excitation. At the same time, many other researchers, such as Harichandran & Wang [4] and Nazmy & Abdel-Ghaffar [5], have been working on the numerical analysis of seismic behavior of long-span bridges subject to such excitation.

Even though this field has become quite attractive to many researchers in the last decades, few experimental studies have been carried out. One of the first test studies was conducted at the University of California, Berkeley, using a single shake table to simulate the 1971 San Fernando earthquake on a bridge-scale model [6]. However, in order to simulate non-uniform excitation, a system of multiple shake tables is required. Very few institutions own more than one shake table [7], but one of them is the University of Rijeka (Faculty of Civil Engineering), owning two biaxial shake tables, which allow conducting experiments on models subject to non-uniform ground motions [8].

An experimental programme with the main objective of examining the dynamic behavior of simply-supported beams to multiple support excitation is here designed and carried out. The practical limitations of conducting such experiments that could be used to validate linear dynamic analysis are discussed, and experimental benchmark cases for six different excitations are given.

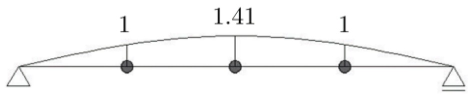
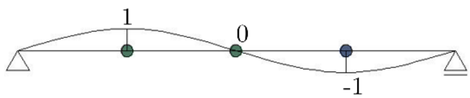
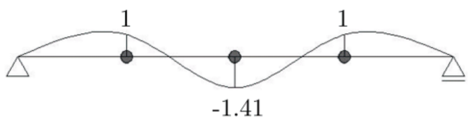
2. Experimental setup and model description

A special experimental setup has been designed to examine the dynamic behavior of small-scale one-dimensional structures with two supports subjected to both uniform and non-uniform excitation.

2.1. Model

A wooden beam of 2.0 m length and a cross section of 46.4 by 12.8 mm is used. Its density is 0.5792 g/cm^3, mass is 688 g, and Young's modulus is 12.78 GPa. For the simply supported case and with weights of 8 N attached to the quarters of its span, its natural frequencies calculated using modal analysis from [2] and corresponding shapes are given in Table 1.

Table 1. *Natural frequencies of the beam model*

Mode	Natural frequency	Shape
1st	18.026 rad/s (2.869 Hz)	
2nd	72.957 rad/s (11.611 Hz)	
3rd	157.188 rad/s (25.017 Hz)	

2.2. Excitation and measuring system

The beam is placed on a unique system of two biaxial shake tables Quanser STI-III (Figure 1), which are used to excite it. All the measurements are obtained by means of accelerometers connected to the shake tables and a non-contact optical measuring system GOM Aramis and Pontos (version 6.3 and 8.0). All the experiments are filmed with full resolution (2400x1728 pixels) and 160 frames-per-second (fps), with measurement accuracy of 0.2 mm.

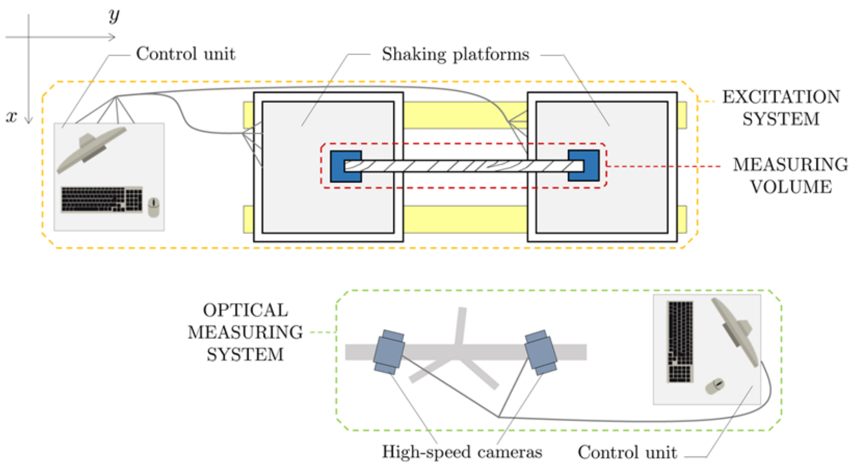


Figure 1. Top view of the experimental set-up

3. Results and discussion

A series of preliminary tests on a SDOF cantilever beam is performed at the beginning in order to measure the properties of the system, as well as to detect the optimal amplitude of the excitation function that will trigger only linear response.

The dynamic response of the MDOF beam described in Section 2 is analyzed for six different excitation conditions: three synchronous excitations designed to excite the symmetric modes of vibration, and three asynchronous excitations designed to test the asymmetric modes of vibration.

3.1. SDOF cantilever beam tests

The purpose of the tests carried out on a SDOF cantilever beam (Figure 2) is to determine the damping ratio and the boundary values of excitation amplitudes needed to avoid the non-linear effects in the response.

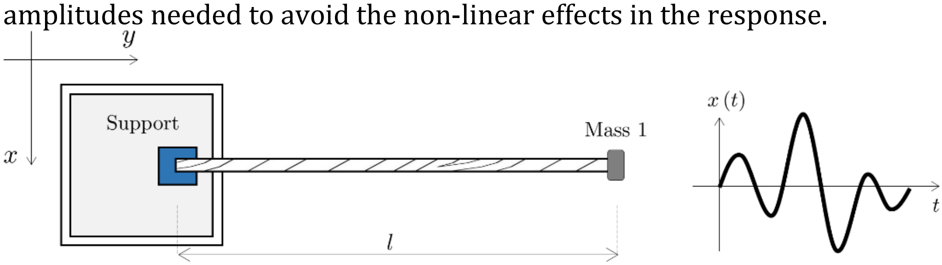


Figure 2. Top view of the experimental set-up with a SDOF beam

3.1.1. Damping ratio determined from free oscillation tests

The system is initially moved from the equilibrium position and released to oscillate with its natural frequency (Figure 3).

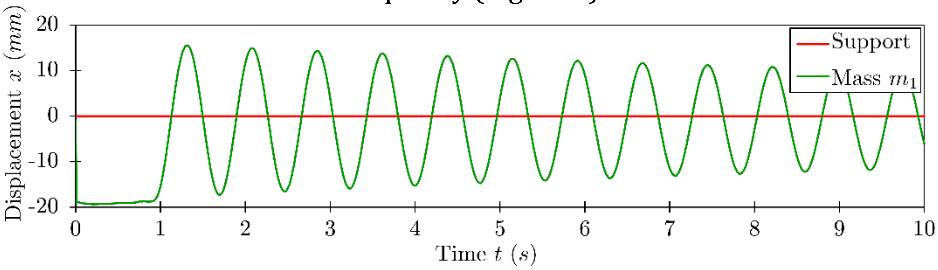


Figure 3. Experimentally obtained free oscillations of a SDOF beam

From this displacement-time graph (Figure 3) the damping ratio is calculated using the logarithmic decrement [2]

$$\zeta = \frac{\ln\left(\dfrac{A_i}{A_{i+1}}\right)}{\sqrt{4\pi^2 + \ln^2\left(\dfrac{A_i}{A_{i+1}}\right)}} = 0.006326\,, \tag{1}$$

where A_i is the displacement of the i^{th} peak (i^{th} amplitude). If the system behaves linearly during the motion, the logarithmic decrement will remain constant throughout entire motion, as well as the period of oscillation.

A series of such free oscillation tests with varying initial displacement amplitude has shown that the amplitudes with which the beam is excited should be very small if the system is expected to behave linearly, preferably around one centimeter in this case.

3.1.2. Optimal excitation amplitude determined from forced oscillation tests

One of the main challenges in this research has been choosing the appropriate amplitude of excitation which will not trigger non-linear effects. Even though the amplitudes of excitation need to be small enough so that only linear effects are triggered, they have to be large enough so that the experimental equipment can simulate the excitation function satisfactorily. The shake tables have linear encoders, which measure the actual displacements. It has also been noted that, for very small excitation amplitudes, this displacement-time history output does not ideally coincide with the one that is assigned as input. This has led to a series of harmonic support excitation tests aimed at detecting the minimum value of the excitation amplitude, which results in a satisfactory output (real) excitation.

The value of 0.6 mm roughly satisfies these requirements. Both the input and output (real) excitation function are presented below.

3.2. MDOF simply supported beam tests

The main experimental set-up consists of the previously described beam with weights of 8N placed into the quarters of the span simply-supported on the two shake tables (Figure 4). The calculated values for the first and the second natural frequency of the beam are $\omega_1 = 18.026\ rad/s$ and $\omega_2 = 72.957\ rad/s$ (Table 1). The experimental values of these frequencies have been obtained by observing the response ratio of the beam and detecting the frequencies which caused peak response ratio of the symmetric (1st) and asymmetric (2nd) mode. These values are $\omega_{exp,1} = 18.850\ rad/s$ ($f_{exp,1} = 3\ Hz$) and $\omega_{exp,2} = 72.257\ rad/s$ ($f_{exp,2} = 11.5\ Hz$). The chosen excitation amplitude is $u_0 = 1\ mm$ and the damping ratio (calculated from the SDOF experiments) is $\zeta = 0.006326$.

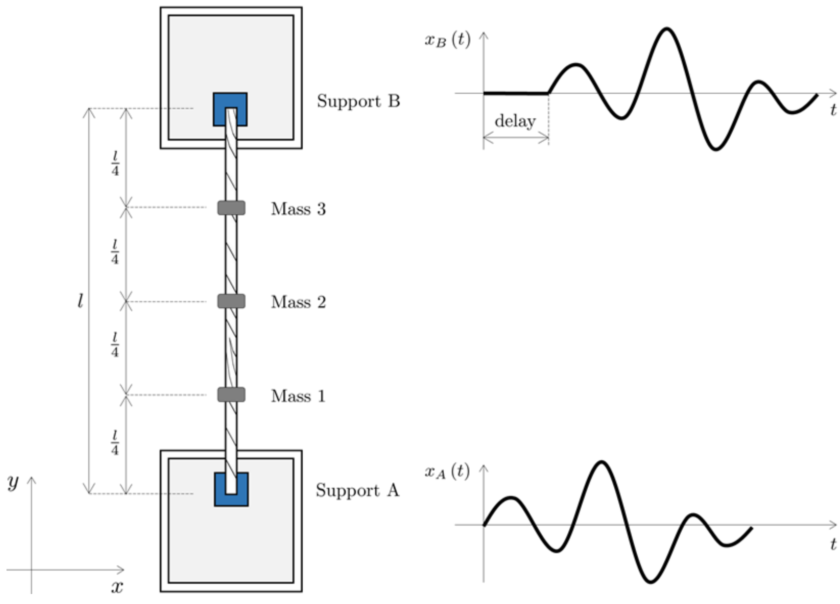


Figure 4. Top view of the experimental set-up with MDOF beam

3.2.1 Synchronous harmonic excitation

The harmonic excitation used to excite the beam is a sinusoidal displacement function

$$u_g(t) = u_0 \sin(\omega t), \tag{2}$$

where u_0 and ω are the amplitude and the angular frequency of the displacement function.

Firstly, both of the beam supports are simultaneously excited with a harmonic function with $u_0 = 0.001\ m$ and $\Omega_1 = \omega_{exp,1} = 18.850\ rad/s$. It is expected that the symmetrical mode shapes (the 1st and the 3rd one) would be excited and, in this case, the experiment confirms the theoretical assumptions (Figure 5).

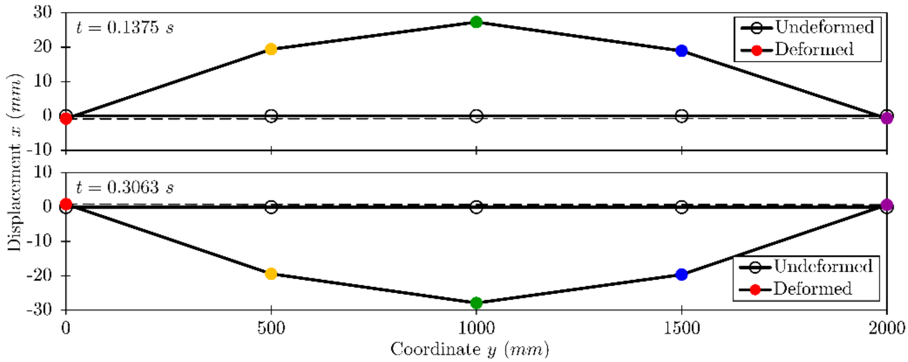


Figure 5. Deformation due to synchronous excitation with $u_0 = 1\ mm$ and $f = 3\ Hz$ ($\omega_{exp,1} = 18.850\ rad/s$, close to 1st resonant frequency)

The dynamic response for such excitation is shown in Figure 6. Clearly, the measured excitation is not identical to the input excitation function. As described in Section 3.1.2, this cannot be controlled more precisely owing to the limitations of the shake tables.

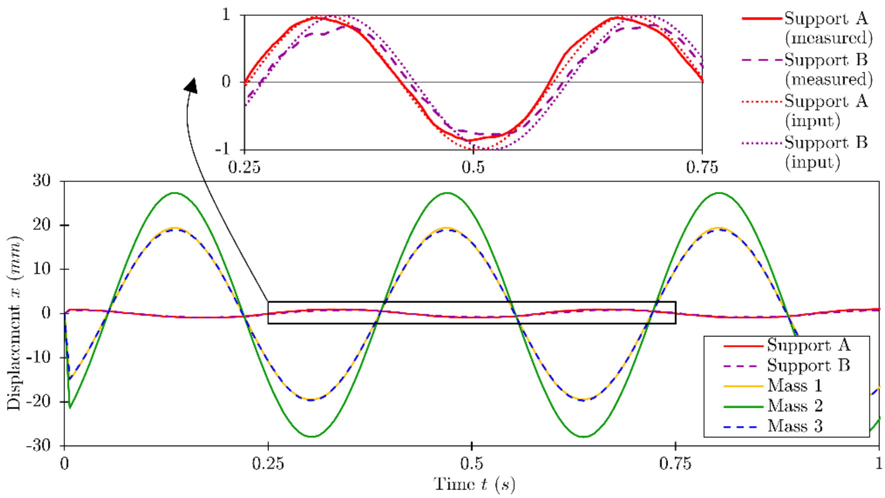


Figure 6. Excitation (top) and response (bottom) due to synchronous excitation with $u_0 = 1\ mm$ and $f = 3\ Hz$

Experimentally, the response ratio reaches the value of $R_d = 28.382$, which is far below the analytically calculated value. It must be added that a small change in the excitation frequency at resonance causes a large change in the response ratio and any present imprecision (such as e.g. in the material properties or excitation frequency) is expected to have a large effect.

Secondly, the experiment is repeated for the case of a synchronous excitation with $u_0 = 0.0005\ m$ and $\Omega_2 = \omega_{exp,2} = 72.257 rad/s$. Ideally, only the symmetrical mode shapes would be excited, even though the excitation frequency is chosen to be as close as possible to the second resonant frequency. The following graphs show that the beam acts slightly different from that and a contribution of the skew-symmetric (2nd) mode of oscillation can very much be noticed (Figure 7). This behavior may be a result of small differences between the excitation simulated with the two shake tables, providing a minor source of asynchronous excitation, which would probably have no effect and go unnoticed if the excitation frequency were not close to the 2nd natural frequency. Moreover, any deviation from symmetry in geometry or material properties is bound to have a pronounced effect for such an excitation frequency.

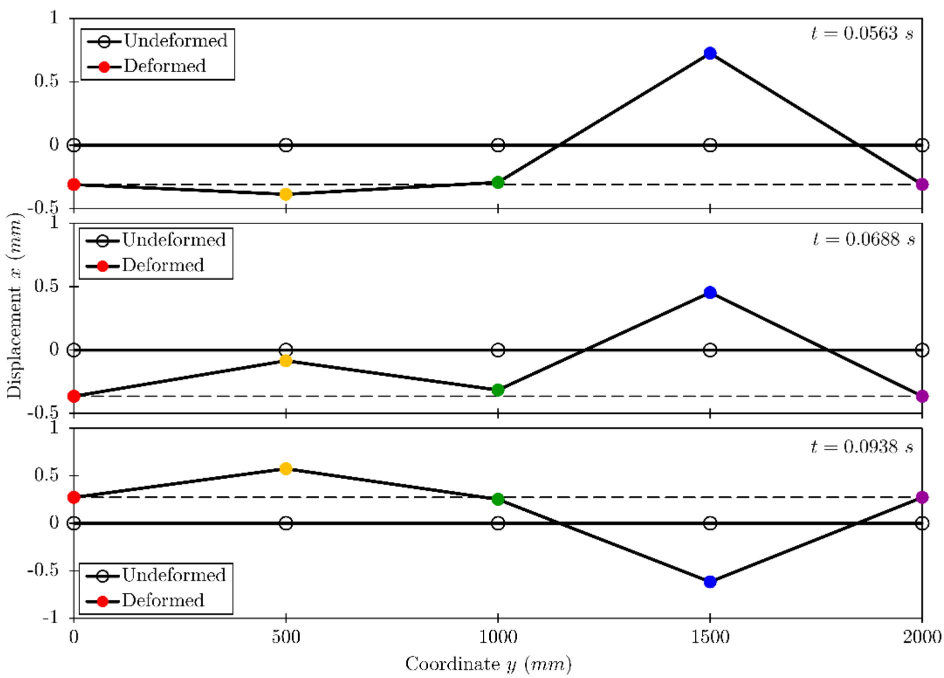


Figure 7. Deformation due to synchronous excitation with $u_0 = 0.5\ mm$ and $f = 11.5\ Hz$ ($\omega_{exp,2} = 72.257 rad/s$, close to 2nd resonant frequency)

The corresponding dynamic response is shown in Figure 8.

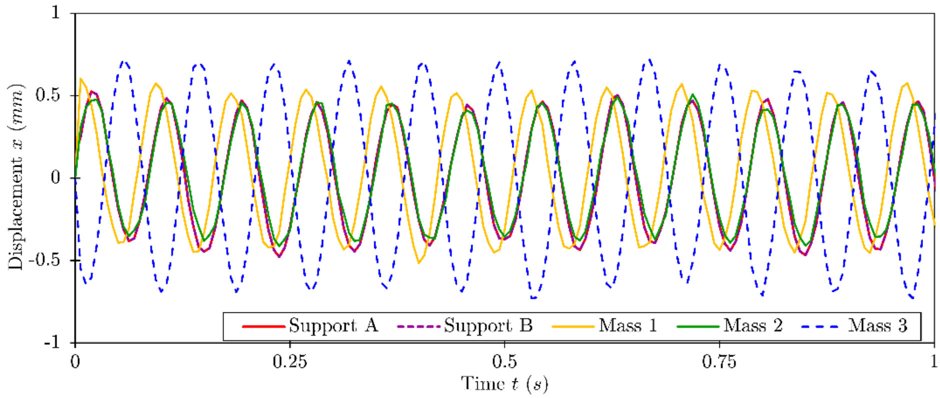


Figure 8. Excitation and response due to synchronous excitation with $u_0 = 0.5\ mm$ and $f = 11.5\ Hz$

3.2.2. Asynchronous harmonic excitation

When the beam is subject to an asynchronous excitation, the supports undergo the same motion function (2), but with the opposite sign. Under such skew-symmetric excitation, the beam is expected to follow the skew-symmetric (2nd) mode shape.

Firstly, the beam is excited with an asynchronous excitation with $u_0 = 0.0005\ m$ and $\Omega_1 = \omega_{exp,1} = 18.850\ rad/s$ ($f = 3\ Hz$). As it can be seen in Figure 9, the beam oscillates with a shape, which resembles the 2nd mode shape. However, we can notice a contribution of a symmetric mode shape from the position of the middle mass.

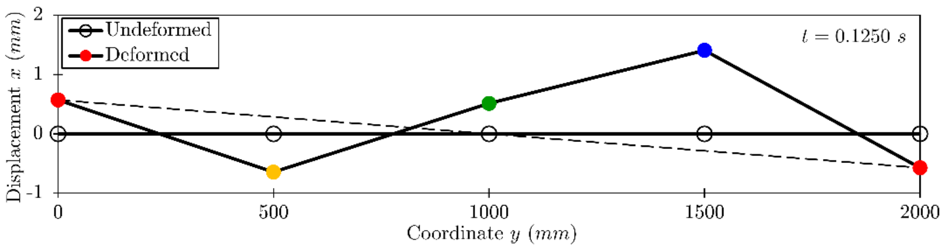


Figure 9. Deformation due to asynchronous excitation with $u_0 = 0.5\ mm$ and $f = 3\ Hz$ (close to 1st resonant frequency)

We attribute such behavior to the same reasons as those provided for the earlier case of the synchronous excitation at a frequency close to the 2nd resonant frequency. This is not expected to happen when the excitation frequency is close to the 2nd natural frequency, which should trigger only skew-symmetric modes.

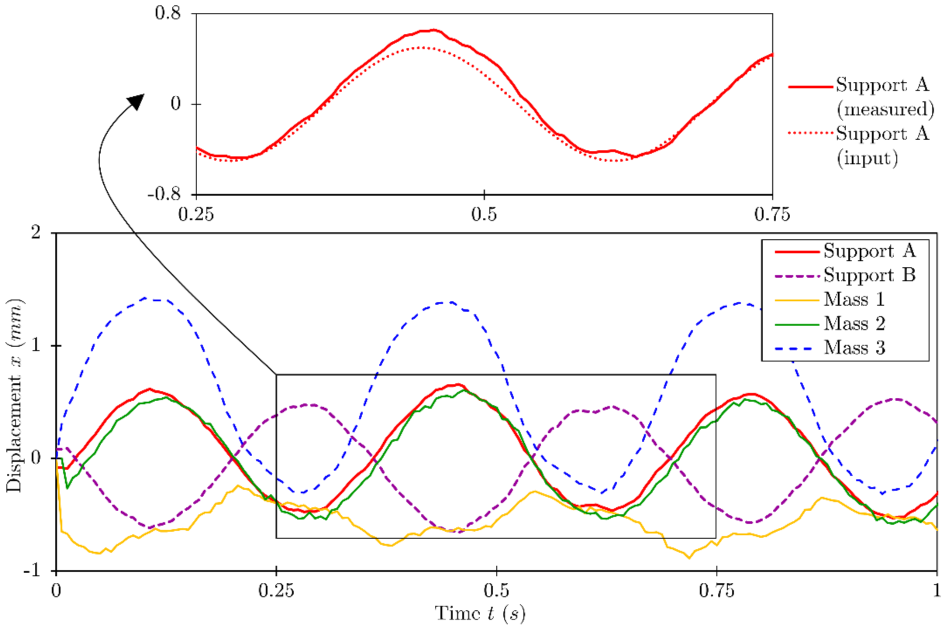


Figure 10. Excitation (top) and response (bottom) due to asynchronous excitation with u_0 *= 0.5 mm* and f *= 3 Hz*

As expected, a better correlation between the expected mode shape and the recorded one is obtained for the asynchronous excitation with $u_0 = 0.012$ *m* and the second resonant frequency, $\Omega_2 = \omega_{exp,2} = 72.257 rad/s$ (Figure 11), where the mode of oscillation is completely skew-symmetric. Note that the amplitude of the excitation function is here much larger than in the previous experiment, so that the modes of vibration may be better observed.

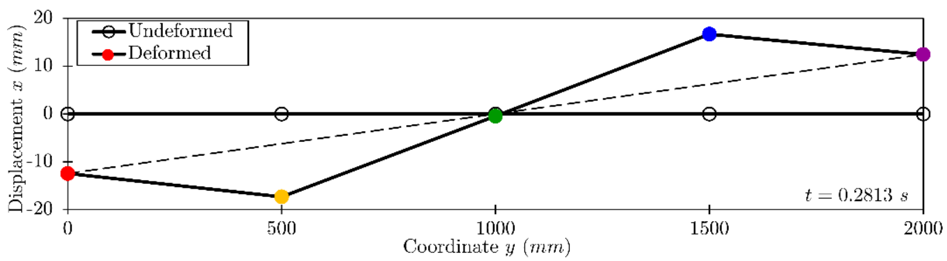


Figure 11. Deformation due to asynchronous excitation with u_0 *= 12 mm* and f *= 11.5 Hz* ($\omega_{exp,2}$ *= 72.257rad/s,* close to 2nd resonant frequency)

This excitation results in a response, which almost ideally follows 2nd mode of oscillation, with the exception of a small oscillation of the middle degree of freedom (mass 2) around its initial position (Figure 12).

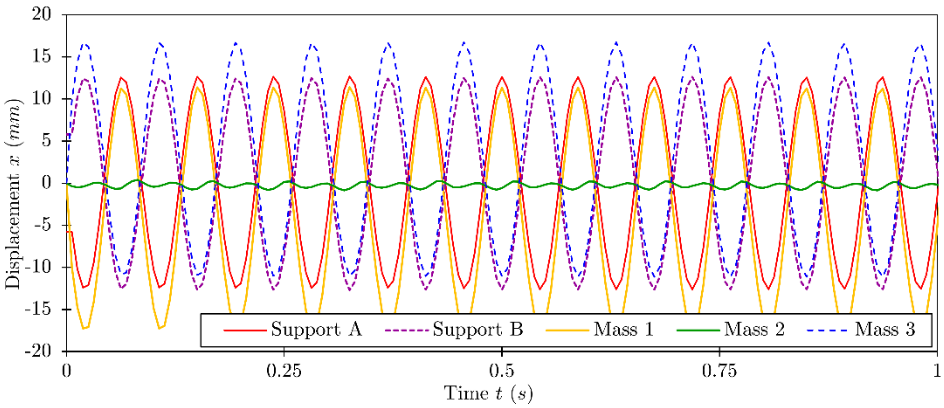


Figure 12. Excitation and response due to asynchronous excitation with u_0 = 12 mm and *f = 11.5 Hz*

3.2.3 Synchronous earthquake excitation

Finally, the behavior of the beam subject to both uniform and non-uniform earthquake excitation was explored and documented as a unique experimental benchmark case of a multiple-support-excitation behavior.

The Northridge 1994 earthquake acceleration record is scaled to be simulated with the available shake tables system. When the earthquake excitation is simulated uniformly with both shake tables, thus exciting both supports simultaneously, the response is largest when the excitation is the largest, after which the oscillations decrease relatively quickly (Figures 13 and 14).

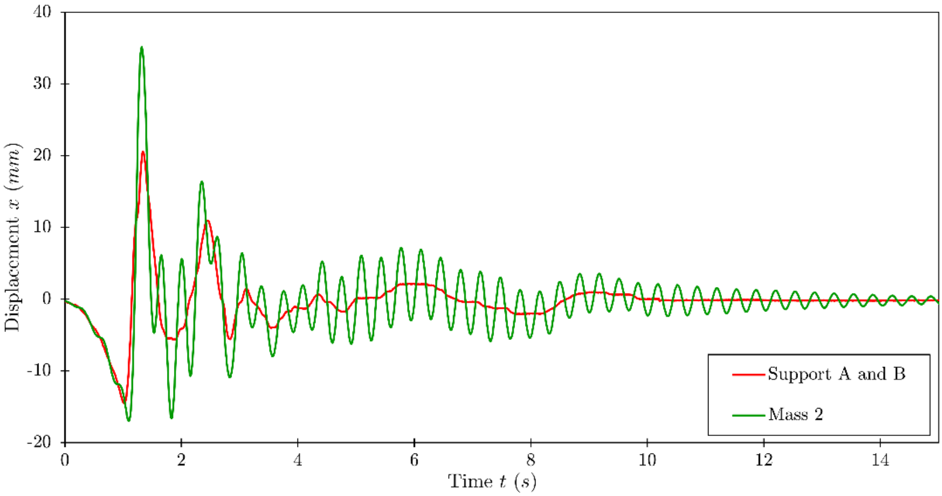


Figure 13. Earthquake excitation and response of the beam

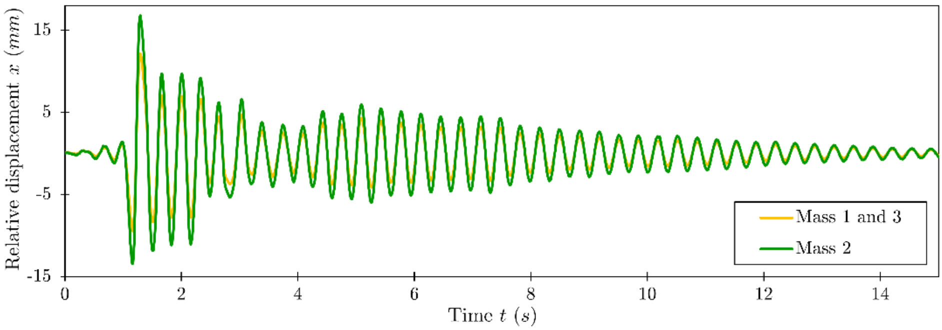


Figure 14. Relative displacements of the DOFs of the beam

3.2.4 Earthquake excitation with delay

Non-uniform earthquake excitation is achieved by again simulating Northridge 1994 earthquake with both tables, but now with a time delay between the beginning of the excitation at table A (support A) and at table B (support B) as $t_d = 1\ s$. The displacements, which occur in this case, are not so large when the excitation is the strongest, but later in time they dissipate more slowly than in case of uniform excitation (Figures 15 and 16). This kind of behavior can represent an alternative dangerous scenario, which is usually not taken into account in the assessment of the dynamic response of structures.

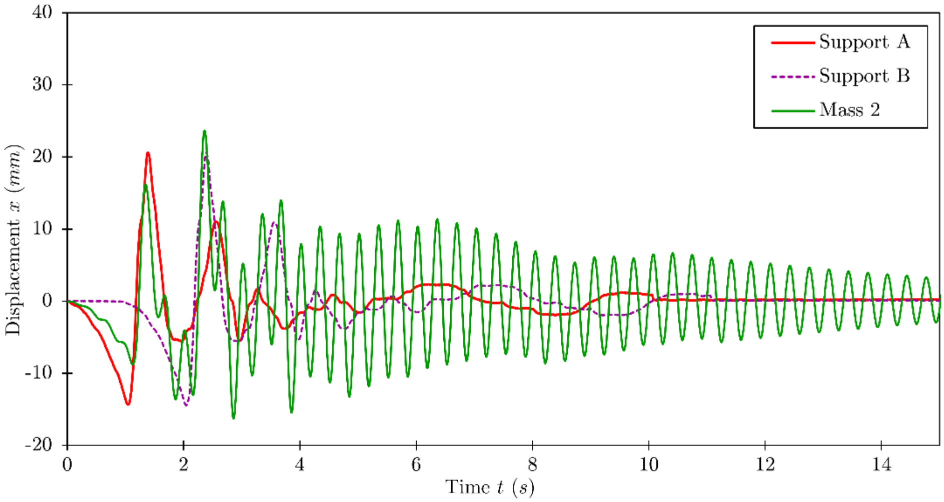


Figure 15. Excitation and response of the beam subject to earthquake excitation with delay

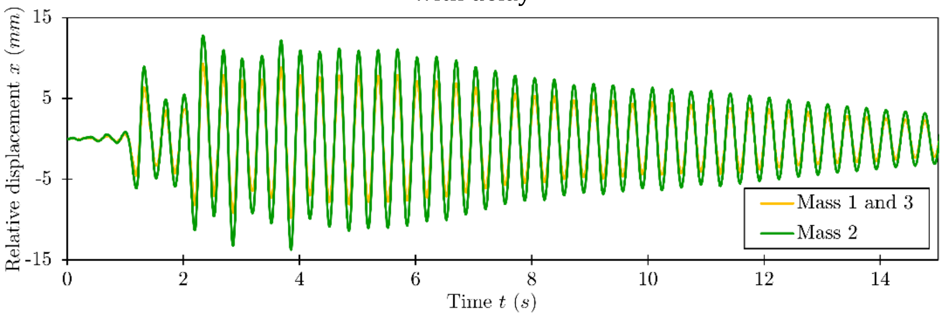


Figure 16. Relative displacements of the DOFs of the beam subject to earthquake excitation with delay

4. Conclusion

The presented experimental analysis confirms the expected dynamic response, which suggests that ideally synchronous excitation triggers synchronous response, while ideally asynchronous excitation triggers asynchronous response. At the same time, this study shows how even minor sources of excitation can trigger significant contribution of asynchronous shapes in the total response of beams if the excitation frequency is close to the natural frequencies of asymmetric modes.

Finally, the dynamic response of beam structure to earthquake excitation shows that the response can be significantly different if different supports are subjected to the same earthquake function but with a certain delay, which simulates real earthquake conditions for long structures.

The concluding remarks open the way for developing a novel approach for seismic design of larger structures greatly affected by the non-uniform earthquake excitation. Furthermore, the presented model tests may be helpful in validation of analytical and numerical procedures aiming to assess the dynamic response of structures subjected to multiple support excitation.

Acknowledgements. *This work has been supported in part by Ministry of Science, Education and Sports of the Republic of Croatia under the project Research Infrastructure for Campus-based Laboratories at the University of Rijeka (RC.2.2.06-0001). The project has been co-funded from the European Fund for Regional Development (ERDF). The results presented in this paper are the results of work done within following projects: Configuration-dependent interpolation in non-linear finite-element analysis of structures (HRZZ-IP-11-2013-1631), supported by Croatian Science Foundation and Testing of slender spatial beam structures with emphasis on model validation (13.05.1.3.06), supported by University of Rijeka.*

Literature

[1] Heuer, R. (2016). *Meccanica dei materiali e delle Strutture*: On Multiple Support Random Excitation of Bridge Structures. Vol. VI, no. 1, pp. 50-57; ISSN: 2035-679X

[2] Chopra, A.K. (1995). *Dynamics of Structures: Theory and Applications to Earthquake Engineering*. Prentice Hall, Englewood Cliffs (New Jersey)

[3] Clough, R.W. & Penzien, J. (2003). *Dynamics of Structures*. Computers and Structures, Inc., Berkeley

[4] Harichandran, R. & Wang, W. (1988). *Response of Simple Beam to Spatially Varying Earthquake Excitation*. Journal of Engineering Mechanics 114(9)

[5] Nazmy, A.S. & Abdel-Ghaffar, A.M. (1992). *Effects of Ground Motion Spatial Variability on the Response of Cable-Stayed Bridges*. John Wiley & Sons Ltd. Vol. 21, Pg. 1-20. Hoboken (New Jersey)

[6] Li, J. et al. (2015). *Shake Table Studies of Seismic Structural Systems of a Taizhou Changjiang Highway Bridge Model*. Journal of Bridge Engineering, ASCE

[7] Yang, C.-Y. et al. (2011). *Shake table Test of Cable-Stayed Bridge Subjected to Non-Uniform Excitation*. Procedia Engineering 14:931-938

[8] Cuculić, M. et al. (2016). *Potencijali istraživačkog rada na Građevinskom fakultetu u Rijeci*. Sveučilište u Rijeci, Građevinski fakultet